(E) Remarks

Specification

In new page 4 the passage referring to the GOHFER software has been corrected. New page 22 includes a description of the index notation. In new page 25 the term b_j is now replaced by the term b_i as in equation (1).

Claims

Claim 1 has been amended to remove to recite that the first set of data contains time history of fluid and proppant volumes, fluid and proppant properties and geological properties. Since proppant is not systematically added (this is in particular the case during the so-called pad-phase at the beginning of most fracturing treatment), it has been further précised that the proppant properties are only included in the set of data if the fluid effectively contains proppant. Claims 6-9 and 12 have been similarly amended.

Claim 3 has been rewritten as an independent claim.

Claim 8 has been further amended to remove the reference to real-time monitoring.

Information Disclosure Statement

Applicant believes to have made an objective description of the possibility of the FracCADE software, as it had been developed at the time of the invention.

In the attached second affidavit, Mr. Eduard Siebrits has prepared a document illustrating the P3D and PL3D models, which details what was the inventors understanding of the common knowledge and shows the main differences between the P3D and the PL3D models.

Claim rejections – 35 USC §112

Reference is made to the paragraph numbering of the Final Office Action.

- **Rejection following paragraph 5.1**

An amendment to page 22 of the specification is hereby proposed to explain that the index notation is used.

Details about the index notation can be found for instance in " Boundary Element Methods In Solid Mechanic, by S.L. Crouch and A.M. Starfield, first published in 1983, with copies of pages

8-9 and 17-18. Applicant is respectfully apologizing for the omission of paper with its last response. The submission of this paper is hereby done to show that the index notation is well known and that consequently, the proposed amendment does not add new matter to the specification.

- **Rejection following paragraph 5.2**

It is respectfully noted that Applicant has made the appropriate correction in the former response by amending pages 24 and 25 so that the roots alpha are now effectively layer-dependent. To avoid any misunderstanding, substitutes pages 24 and 25 are resubmitted with the present paper (with page 25 incorporating new modifications, first submitted in the present paper, as indicated in the next paragraph).

- **Rejection following paragraph 5.3**

A new amendment of page 25 is proposed where the term b_j is replaced by the term b_i as in equation (1) is proposed.

- **Rejection following paragraph 5.4**

The phrase "at least one of the following" has been removed from claim 1.

- **Rejection following paragraph 5.5**

In claim 8, the term "in real-time" has been removed from the preamble, as it has been previously removed from paragraph (h).

- **Rejection following paragraph 7**

Step (a) of claim 1 has been revised to provide proper antecedent basis for the limitation "the equilibrium equations" in step (c).

__Claim rejections – 35 USC §103__

Claim 1 was rejected under 35 USC §103 as being unpatentable over GOHFER in view of Linkov. Applicant respectfully disagrees.

First, as mentioned in our previous response, the author of the GOHFER model was stating that his model was applicable to multi-layered formations and therefore one skilled in the art of modeling fracturing treatment had no reason to modify that model to add a function that was arguably already included. In the attached second affidavit Mr. Eduard Siebrits provides physical evidence of the defaults of the GOHFER model. The reference to the GOHFER model on page 4 has been slightly modified to avoid the apparent contradiction between the specification and the affidavits.

Moreover, once the author of the present invention found the actual limitations of the GOHFER model and contemplated implementing the Linkov model, they were told by Mr. Savitski, one of the co-authors of the Linkov paper that this model was not applicable for the case of cavities or cracks intersecting the layers boundaries (to be noted that these cracks are not damages, as the Examiner's note in paragraph 9 may suggest but a model of the actual hydraulic fracture created by injection of fluid under pressure during the hydraulic fracture treatment. The Linkov model is purely theoretical and to the best knowledge of the inventors, has never been implemented. So the inventors of the present invention could only rely on the comments made by Mr. Savitski (and reported in the first affidavit written by Mr. Eduard Siebrits).

Applicants submit that this response addresses all of the issues raised in the official action respectfully request reconsideration and that a timely Notice of Allowance be issued in this case.

It is believed fees are due for this reply. Should additional fees or refunds be due, the Commissioner is authorized to charge or credit any necessary fee to Deposit Account No. 04-1579(56.0428).

Respectfully submitted,

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Limited Recognition
Agent for Applicants

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fracture simulators. Methods employing PL3D aim at accurately ~~take~~ taking into account geologic layers. One ~~such~~ program presented as PL3D, known commercially as GOHFER (GOHFER is believed to be a trademark of Stim-Lab and the Marathon Oil Company), provides grid oriented hydraulic fracture replication capabilities. This grid oriented program, and its mode of operation, is seen in Figure 7. As the front of the fracture moves forward, calculations are made in which each individual grid is either "on" or "off" depending upon whether or not more than half of the individual grid is "covered" by the advancing fracture as it moves outward from the wellbore. If more than one-half of the grid element is covered, then the element is estimated to be fully active. The disadvantage of this system of estimating fracture growth is that it produces too much numerical noise at the fracture tip, and hence in the output data.

Other PL3D methods of simulating fractures include the TerraFrac three dimensional fracturing simulator (TerraFrac is a trademark of the TerraTek Company). This simulator operates as seen in Figure 8, using estimates that are based upon a method of a moving mesh. This method shows less noise than the GOHFER method, because it uses triangle

width and fracture pressure on each active element. A complete description of the process of the propagation of a hydraulic fracture is thus obtained. Solutions of the multi-layer equilibrium equations are provided. A three-dimensional body is assumed. The theory also applies to the two-dimensional cases (plane strain, plane stress, antiplane strain). The method provides an efficient way of determining the solution to the equilibrium equations:

$$\sigma_{ij,j} + b_i = 0 \tag{1}$$

for a transversely isotropic elastic medium with a stress strain relationship given by:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

In Equation (1) and (2) the index notation is used. In this standard notation, a repeated literal index in any term of an expression implies summation. Therefore, in equation (1) $s_{ij,j}$ means

$\sigma_{ij,j} = \sigma_{i1,1} + \sigma_{i2,2} + \sigma_{i3,3}$ *since it is assumed a 3D body.*

The comma preceding an index denotes partial differentiation with respect to that variable represented by that index.; $u_{i,j}$ thus denotes ? u_i / ?x_j . The notation used in Equation (1) is consequently a shortcut for describing the following equations:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

In the case of a transversely isotropic three-dimensional elastic medium, there are five independent

$$L(C_{ijkl})\begin{bmatrix}\hat{u}_x\\ \hat{u}_y\\ \hat{u}_z\end{bmatrix}=\begin{bmatrix}\hat{b}_x\\ \hat{b}_y\\ \hat{b}_z\end{bmatrix} \tag{5}$$

For a layered material, there is a system of differential equations of the form (5) for each layer, each of whose coefficients are determined by the material properties of the layer. It is possible to solve the system of differential equations for a typical layer *l* to obtain the general solution to the *r* th displacement components in the form:

$$\hat{u}_r^l=\sum_j d_{jr}^l e^{\alpha_j^l kz} A_j^l(k) \tag{6}$$

where $k=\sqrt{m^2+n^2}$

In the case of repeated roots of the characteristic equation associated with (5), which occurs for the important case of isotropic layers, the system (5) has the general solution:

$$\hat{u}^l_r = \sum_j \left(d^l_{jr} + f^l_{jr} z\right) e^{\alpha^l_j kz} A^l_j(k) \qquad (7)$$

Here d^l_{jr} and f^l_{jr} are constants that depend on the material constants of the layer, the α^l_j are the roots of the characteristic equation for the system of ordinary differential equations, and the $A^l_j(k)$ are free parameters of the solution that are determined by the forcing terms *bi* in (1) and the interface conditions prescribed at the boundary between each of the layers (e.g. bonded, frictionless, etc.).

Substituting these displacement components into the stress strain law (2), we can obtain the corresponding stress components: $\hat{\sigma}_{xx}$, $\hat{\sigma}_{yy}$, $\hat{\sigma}_{zz}$, $\hat{\sigma}_{xy}$, $\hat{\sigma}_{xz}$, and $\hat{\sigma}_{yz}$, which can be expressed in the form:

$$\hat{\sigma}^l_{pq} = \sum_j \left(s^l_{jpq}\right) e^{\alpha^l_j kz} A^l_j(k) \qquad (8)$$

In the case of repeated roots the stress components assume the form:

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BOUNDARY ELEMENT METHODS IN SOLID MECHANICS

with applications in rock mechanics and geological engineering

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$\sigma_{ij,j}$ and σ_{ij}

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2 Review of linear elasticity

2.1 Introduction

The concepts of stress, strain and linear elasticity are basic to the boundary element methods discussed in this book. We will assume that the reader is familiar with these concepts as developed, for example, in a text on deformable body mechanics. In order to set our notation and sign conventions, however, we will review these topics in this chapter.

2.2 Stress

The concept of stress is used to specify the way in which forces are transmitted through a solid, continuous body. If we imagine an isolated, prismatic element of the body, we can consider that certain forces are exerted on it by the surrounding material. If the element is small enough, these forces are distributed more or less uniformly over each of its faces. The **stress** on any particular face is then defined as the resultant force divided by the area. Stress therefore has units of Newtons per square meter (N/m^2).

Stresses on co-ordinate planes; stress tensor

The stresses acting on planes whose normals are parallel to the co-ordinate axes are known as the components of the **stress tensor**. These stresses are identified in a particular way, illustrated in Figure 2.1. Collectively, the stress components shown in the figure can be denoted by the symbol σ_{ij}. The understanding then is that i and j may be either x, y or z, sometimes represented as x_1, x_2 and x_3 or even simply 1, 2 and 3. The first index (i) refers to the direction of the *normal* to the plane on which the stress acts. The second index (j) refers to the direction of the stress component. Thus, for example, the component σ_{yz} is parallel to the z direction and acts on a plane whose normal is parallel to the y direction.

Sign convention

The sign convention used throughout this book is as follows: the

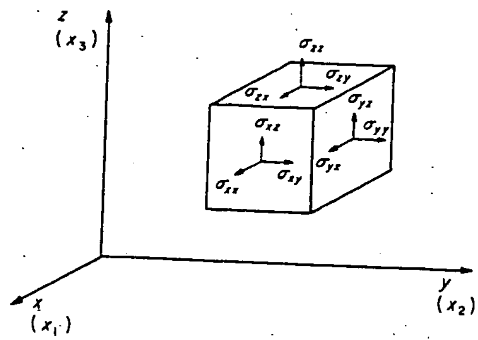

Figure 2.1 Components of the stress tensor.

component σ_{ij} is positive if it acts in the *positive* (or *negative*) j direction on a plane whose outward normal points in the *positive* (or *negative*) i direction. All of the stress components illustrated in Figure 2.1 are positive under this definition. This sign convention means that the stresses normal to the faces of a prismatic element (i.e. σ_{xx}, σ_{yy} and σ_{zz}) are positive if they are tensile.

Equilibrium equations

The components of the stress tensor at various points in a body cannot be specified arbitrarily. The requirement that the body be in equilibrium imposes certain conditions on these components. First, one finds from moment equilibrium conditions that the stress tensor is symmetric, i.e.

$$\sigma_{ij}=\sigma_{ji} \qquad (i, j=1, 2 \text{ or } 3) \tag{2.2.1}$$

This equation says, for example, that if $i=x$ (or 1) and $j=z$ (or 3) then

$$\sigma_{xz}=\sigma_{zx} \qquad (\text{or } \sigma_{13}=\sigma_{31}).$$

Second, it can be shown from force equilibrium considerations that the components of the stress tensor must satisfy the equations

$\sigma_{ij,j}+b_i=0$

$\left[\frac{N}{m^3}\right]$

$$\begin{aligned}
&\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}+\beta_x=0\\
&\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{zy}}{\partial z}+\beta_y=0\\
&\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+\beta_z=0
\end{aligned} \tag{2.2.2}$$

$$\sigma_{zz}=\frac{2G}{1-2\nu}\left[(1-\nu)\ e_{zz}+\nu(e_{xx}+e_{yy})\right]$$

$$\sigma_{xy}=2Ge_{xy} \qquad \sigma_{xz}=2Ge_{xz} \qquad \sigma_{yz}=2Ge_{yz}$$

2.5 Index notation

We have already introduced the practice of replacing the subscripts or indices x, y and z by the numbers 1, 2 and 3. Two further conventions in the use of index notation enable one to write the equations of elasticity in a compact form. The first is that *a repeated literal index in any term of an expression implies summation.* For example, by this 'summation convention' we understand $\sigma_{ij}\, n_j$ to mean

$$\sigma_{ij}\, n_j=\sum_{j=1}^{3} \sigma_{ij}\, n_j=\sigma_{i1} n_1+\sigma_{i2} n_2+\sigma_{i3}\, n_3$$

because the index j is repeated. The second convention is that *a comma preceding an index denotes partial differentiation with respect to the variable represented by that index.* Thus, for instance, $u_{i,j}$ denotes $\partial u_i/\partial x_j$ and $u_{i,jk}$ denotes $\partial^2 u_i/\partial x_j \partial x_k$. Combining the two conventions, we see that $u_{i,jj}$ is a compact way of writing the expression

$$\frac{\partial^2 u_i}{\partial x_1^2}+\frac{\partial^2 u_i}{\partial x_2^2}+\frac{\partial^2 u_i}{\partial x_3^2}$$

Using these two conventions, the important equations introduced so far can be summarized succinctly. The reader can confirm that the equilibrium equations (2.2.2) can be written as

$$\sigma_{ji,j}+\beta_i=0, \tag{2.5.1}$$

the components of the traction vector (2.2.7) are

$$t_i=\sigma_{ji}\, n_j \tag{2.5.2}$$

and the strain tensor (2.3.1) is defined by the relation

$$e_{ij}=\tfrac{1}{2}(u_{i,j}+u_{j,i}) \tag{2.5.3}$$

where, in each case, indices i and j are 1, 2 or 3 in three dimensions and 1 or 2 in two dimensions.

The stress–strain relations can also be written in index notation with the aid of a special symbol called the Kronecker delta, which is defined as follows:

$$\delta_{ij} = \begin{cases} 1 \text{ if } i=j \\ 0 \text{ if } i \neq j \end{cases} \tag{2.5.4}$$

The stress–strain relations can then be put in the form

$$e_{ij} = \frac{1}{2G}\left[\sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{kk}\,\delta_{ij}\right] \tag{2.5.5}$$

and

$$\sigma_{ij} = 2G\left[e_{ij} + \frac{\nu}{1-2\nu}e_{kk}\,\delta_{ij}\right] \tag{2.5.6}$$

It is a simple exercise to verify that (2.5.5) is equivalent to (2.4.1) and that (2.5.6) is equivalent to (2.4.3).

We will find it convenient in this book to use both index notation and unabridged notation where x, y and z refer to the individual cartesian co-ordinates. It should be clear in context whether or not index notation is being used. The letters x, y and z will never be used as subscripts in index notation. Thus, for example, σ_{xx} will always represent just one component, the x–x component, of the stress tensor, whereas σ_{kk} will represent (by the summation convention) $\sigma_{11}+\sigma_{22}+\sigma_{33}$ or, in two dimensions, $\sigma_{11}+\sigma_{22}$.

2.6 Plane stress and plane strain

Many engineering problems in elasticity are essentially two-dimensional in nature and can be classified as problems in either plane stress or plane strain. Plane stress means that the stresses are restricted to a single plane, say the x, y plane. In this case we define plane stress by the conditions $\sigma_{zz}=\sigma_{xz}=\sigma_{yz}=0$ (see Sec. 2.2). Plane strain means that the strains are restricted to a single plane. We specify a state of plane strain for the x, y plane by the conditions $e_{zz}=e_{xz}=e_{yz}=0$ (see Sec. 2.3).

Stress–strain relations for plane stress and plane strain can be written by using the above definitions in conjunction with the general three-dimensional stress–strain relations given in Section 2.4. For plane stress, we set $\sigma_{zz}=\sigma_{xz}=\sigma_{yz}=0$ in (2.4.1) and write